Exercise 4: Thermo-mechanical analysis on an Energy Wall

A shallow tunnel has to be constructed in an urban environment. The possibility of thermal activation is being considered by the owner of the site, hence a preliminary estimation of the thermally-induced mechanical effects has to be evaluated.
The tunnel structure has a rectangular cross section and the heat exchanger pipes should be installed on the walls but not on the slab.

The student is required to analyse the tunnel structure (in terms of bending moment and shear forces) and to calculate the following requirements:

1. To evaluate the lateral earth pressure distribution on the side of the wall.
2. To analyse and solve the structure before the thermal activation.
3. To analyse and solve the structure with thermal loads (cooling: ΔT= -5°C).
4. To analyse and solve the structure with thermal loads (heating: ΔT= +5°C).
5. To sum the effects of points "*2. and 3*". and points "*2. and 4.*" and evaluate the envelope of bending moments and shear forces.
6. To compare and comment the results of point 5.

Materials
Consider a shallow tunnel constructed in a uniform soil deposit with groundwater table (GWT) located 1 m below the surface. The soil is considered to be fully saturated below the GWT and dry above the GWT. The tunnel is made of reinforced concrete. The soil and wall properties are listed in Tables 1 and 2.

Table 1. Soil properties.

	γ'_s	γ'_{dry}	γ'_w	φ'
	[kN/m^3]	[kN/m^3]	[kN/m^3]	deg
Soil	10	15	10	30

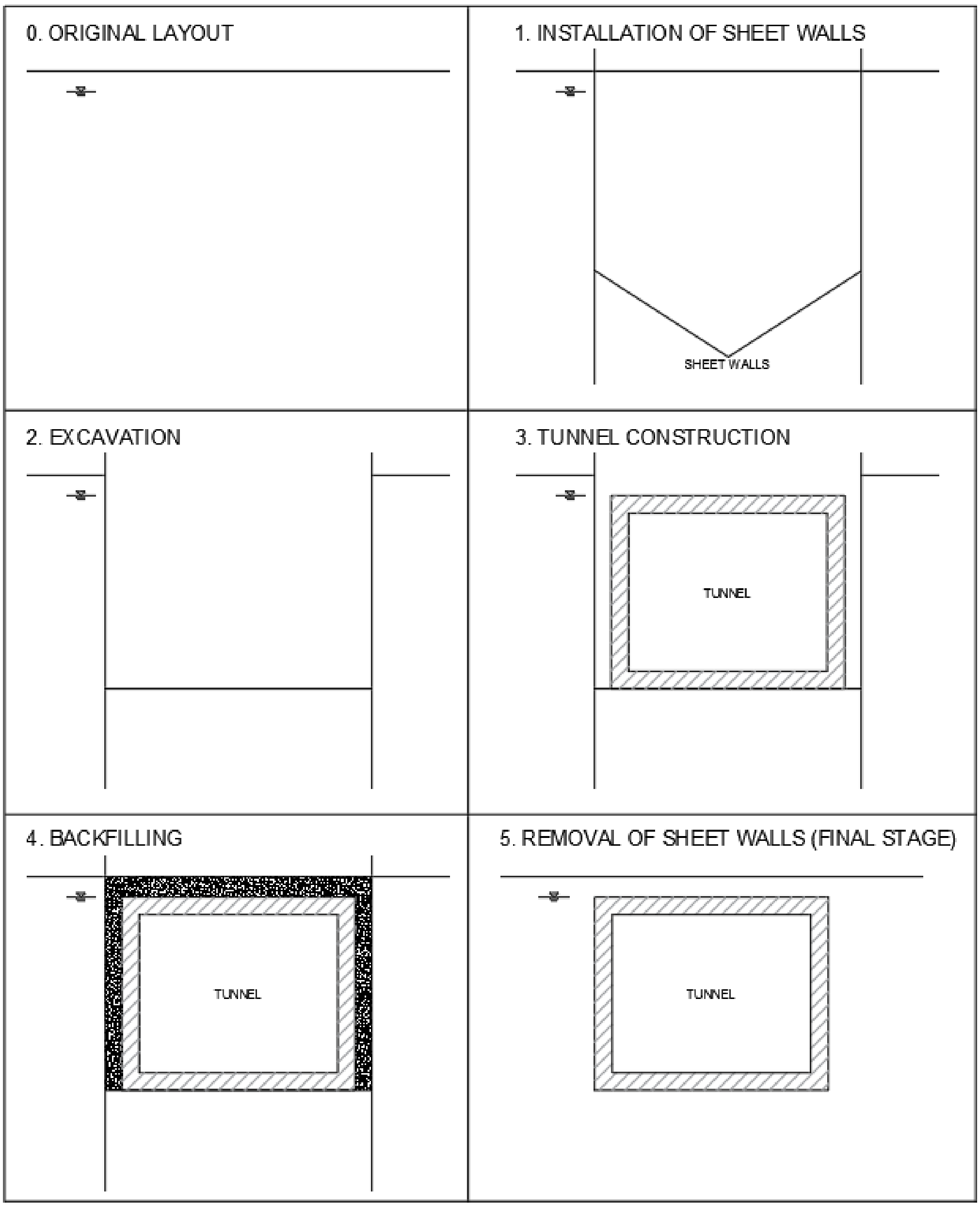
Table 2. Wall properties.

	$E_{concrete}$	$\alpha_{concrete}$
	[MPa]	[με/°C]
Concrete	25000	10

Construction stages

In order to get the final step of construction, a number of stages have to be taken into account and are reported in Figure 1.

Figure 1 Construction stages

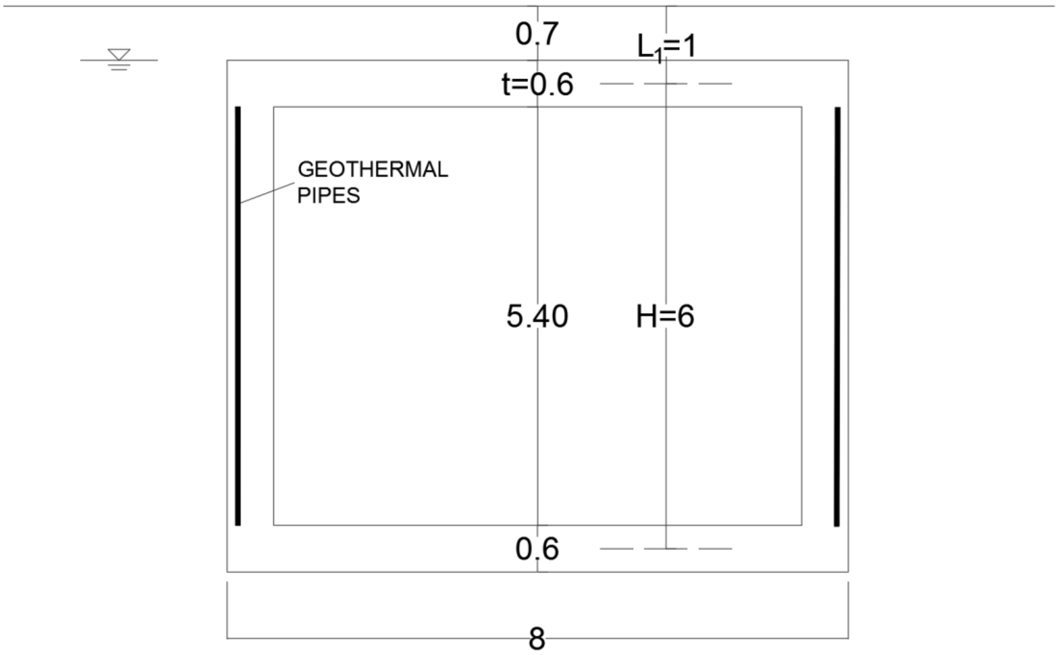


The thermal activation takes place after the end of stage 5. Because of the construction technique, the soil surrounding the tunnel can be considered *at rest* conditions.

Geometry

The final geometry of the tunnel is described in Figure 2.

Figure 2 Geometry of the tunnel. Quotes in meters.



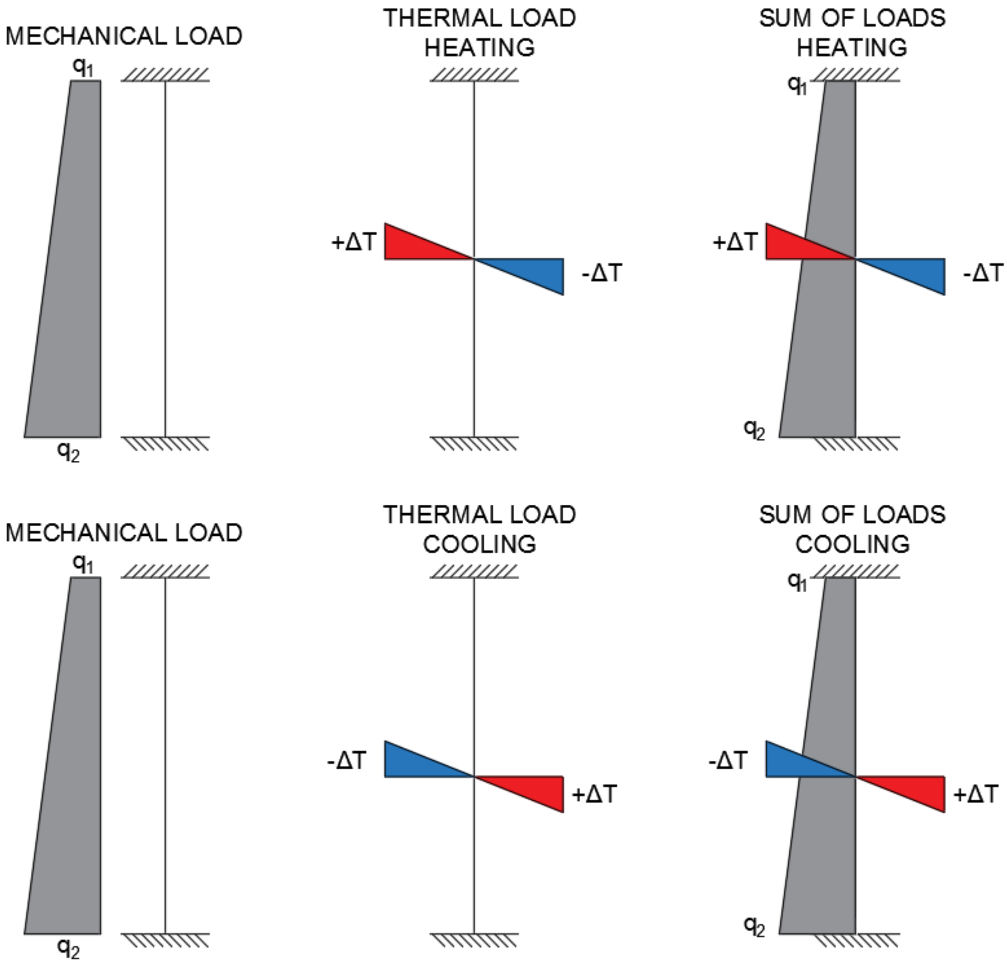
The calculations should be performed with reference to a slice of b=1 m of tunnel.

Table 3. Geometry.

H	**t**	**L_1**	**b**
m	m	m	m
6	0.6	1	1

Due to the construction technique, the walls can be considered as *perfectly clamped* with the slabs at the top and at the bottom, hence the following thermo-mechanical schemes should be used for the resolution of the structure (Figure 3).

Figure 3Thermo-mechanical schemes



Solution

1. Lateral earth pressure

The soil can be considered to be fully saturated below the groundwater table and dry above GWT, hence:

$$k_0 = 1 - \sin(\varphi') = 0.5$$

$$q_1 = k_0 * \gamma_{dry} * L_1 = 7.5 \frac{kN}{m^2}$$

$$q_2 = k_0 * \gamma_{dry} * L_1 + k_0 * \gamma_s' * H + \gamma_w * H = 97.5 \frac{kN}{m^2}$$

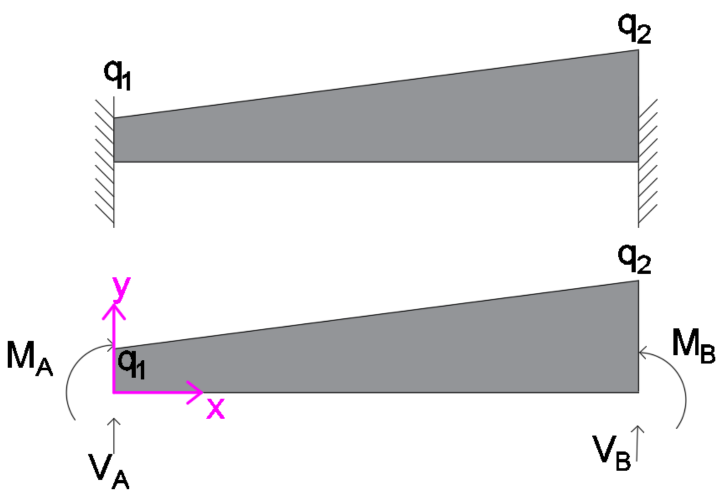
The lateral earth pressure distribution is linear.

2. Solution of the structure with mechanical loads only

The structure is 2 times hyperstatic (axially unloaded).

The schematic view of the structure is:

Figure 4 Schematic view of the structure with mechanical loads



by defining: $k = \frac{q_2 - q_1}{H}$, replacing M_A and V_A with the two hyperstatic unknowns X and Y, respectively, and introducing a reference system x-y as shown in the figure, it is possible to solve the structure using the elastic line method.

$$y''(x) * E_{concrete} I = X + Y * x - \frac{q_1 x^2}{2} - k * \frac{x^3}{6} = M(x)$$

where $I = t * \frac{b^3}{12} = 0.018\, m^4$

integrating once we find the rotation and integrating once again, the displacements:

$$y'(x) * E_{concrete} I = X * x + Y * \frac{x^2}{2} - \frac{q_1 x^3}{6} - k * \frac{x^4}{24} + A$$

$$y(x) * E_{concrete} I = X * \frac{x^2}{2} + Y * \frac{x^3}{6} - \frac{q_1 x^4}{24} - k * \frac{x^5}{120} + A * x + B$$

Boundary conditions

$$y(0) = 0$$

$$y'(0) = 0$$

$$y(H) = 0$$

$$y'(H) = 0$$

which gives:

$$A = 0$$

$$B = 0$$

$$X = -\frac{H^2}{60} * (2 * H * k + 5 * q_1)$$

$$Y = \frac{H}{20} * (3 * H * k + 10 * q_1)$$

Hence the distribution of bending moment M(x) and of shear forces V(x) is:

$$M(x) = X + Y * x - \frac{q_1 x^2}{2} - k * \frac{x^3}{6}$$

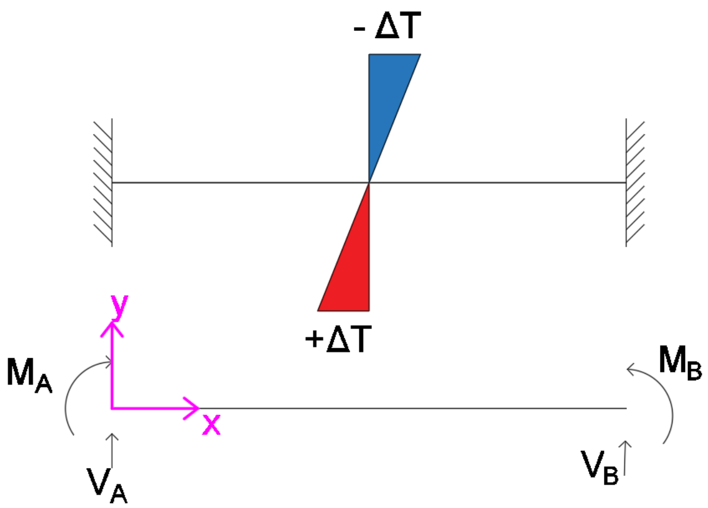
$$V(x) = Y - q_1 * x - k * \frac{x^2}{2}$$

3. and 4. Solution of the structure with thermal loads only

The structure is 2 times hyperstatic (axially unloaded).

The schematic view of the structure is:

Figure 5 Schemativ view of the structure thermically loaded



replacing M_A and V_A with the two hyperstatic unknowns X and Y, respectively, and introducing a reference system x-y as shown in the figure, it is possible to solve the structure using the elastic line method.

$$y''(x) * E_{concrete} I = X + Y * x + \chi^t EI$$

where $\chi^t = 2 * \alpha_{concrete} * \frac{\Delta T}{t}$ is the thermal curvature, and $I = t * \frac{b^3}{12} = 0.018\, m^4$

integrating once we find the rotation and integrating once again, the displacements:

$$y'(x) * E_{concrete} I = X * x + Y * \frac{x^2}{2} + \chi^t EI * x + A$$

$$y(x) * E_{concrete} I = X * \frac{x^2}{2} + Y * \frac{x^3}{6} + \chi^t EI * \frac{x^2}{2} + A * x + B$$

Boundary conditions

$$y(0) = 0$$

$$y'(0) = 0$$

$$y(H) = 0$$

$$y'(H) = 0$$

which gives:

$$A = 0$$

$$B = 0$$

$$X = -\chi^t EI$$

$$Y = 0$$

Hence the distribution of bending moment M(x) and of shear forces V(x) is:

$$M(x) = const = -\chi^t EI$$

$$V(x) = const = 0$$

5. **Comparison**

MECHANICAL LOAD + HEATING

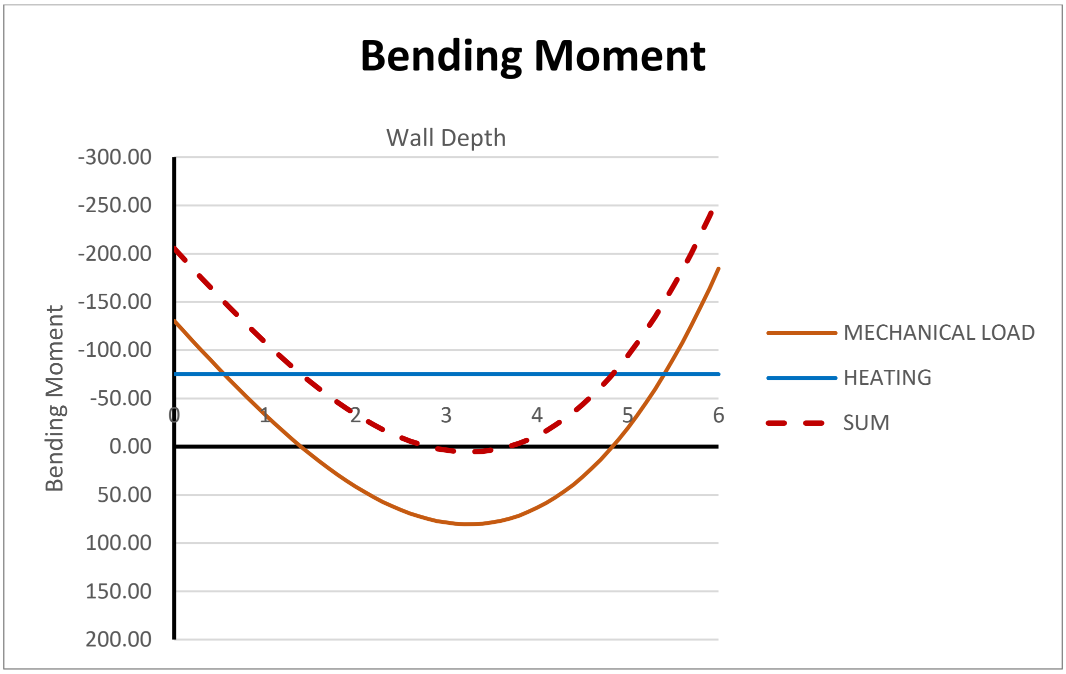
$$M_A = X = -205.5\ kN * m$$

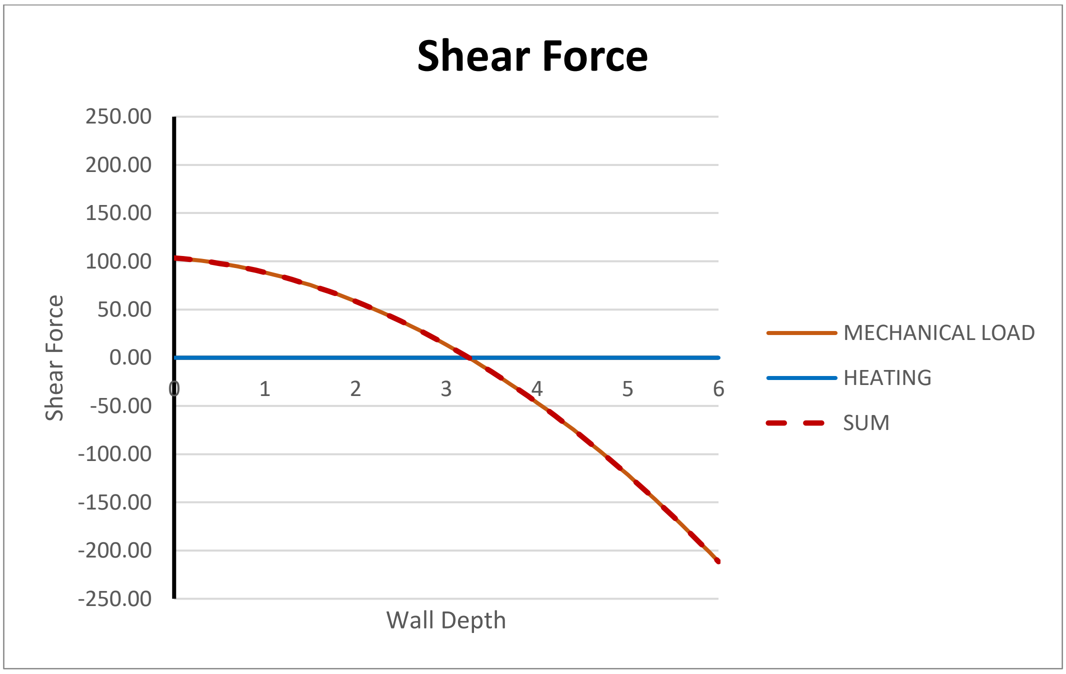
$$V_A = Y = 103.5\ kN$$

$$V_B = 211.5\ kN$$

$$M_B = -259.5\ kN * m$$

$$\chi^t = -0.000167$$

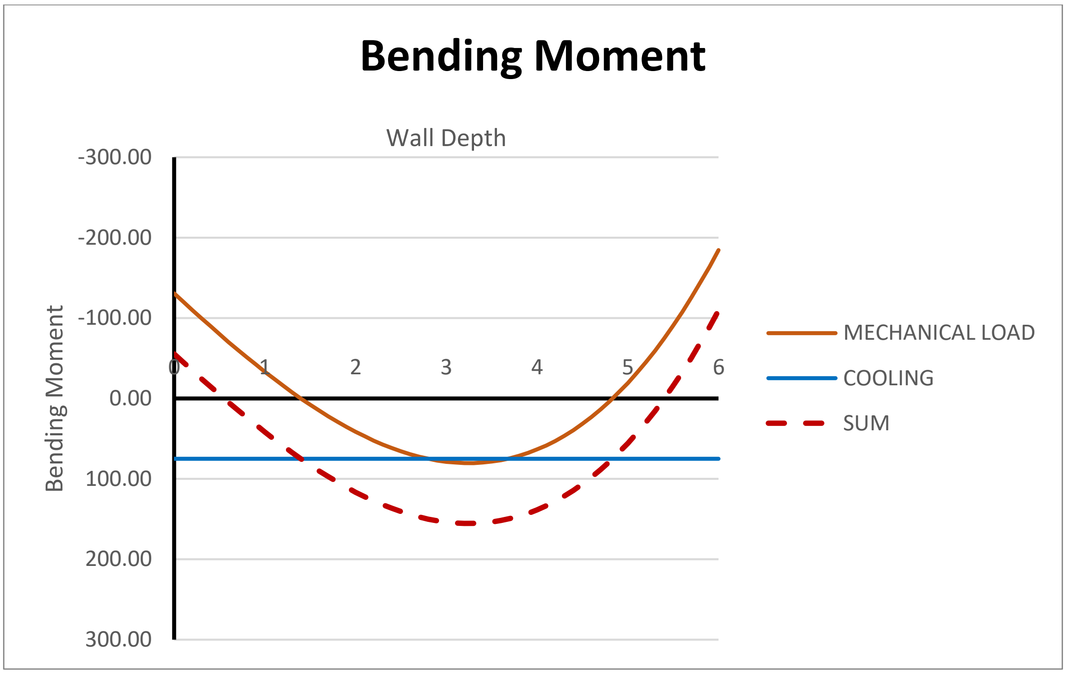


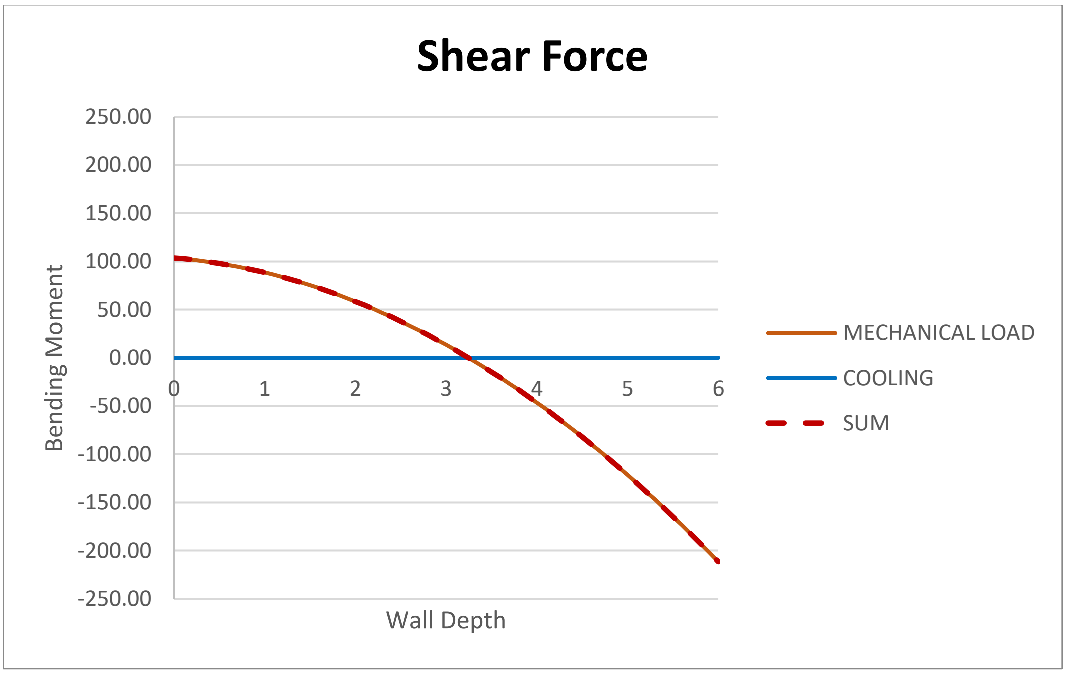


The temperature distribution during heating contributes to increase the positive bending moment M^{-}_{max}.

MECHANICAL LOAD + COOLING

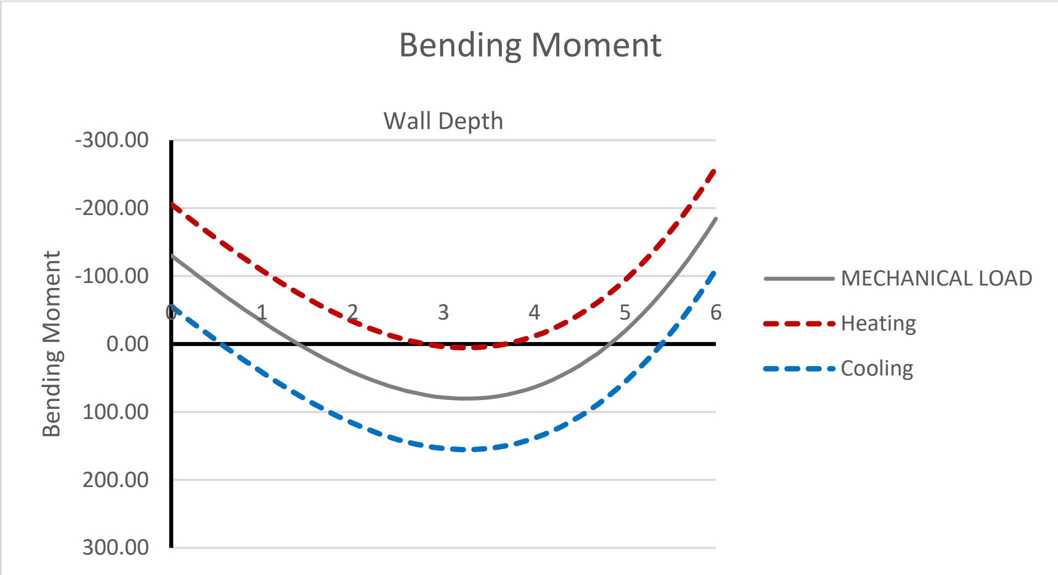
$$\chi^t = 0.000167$$

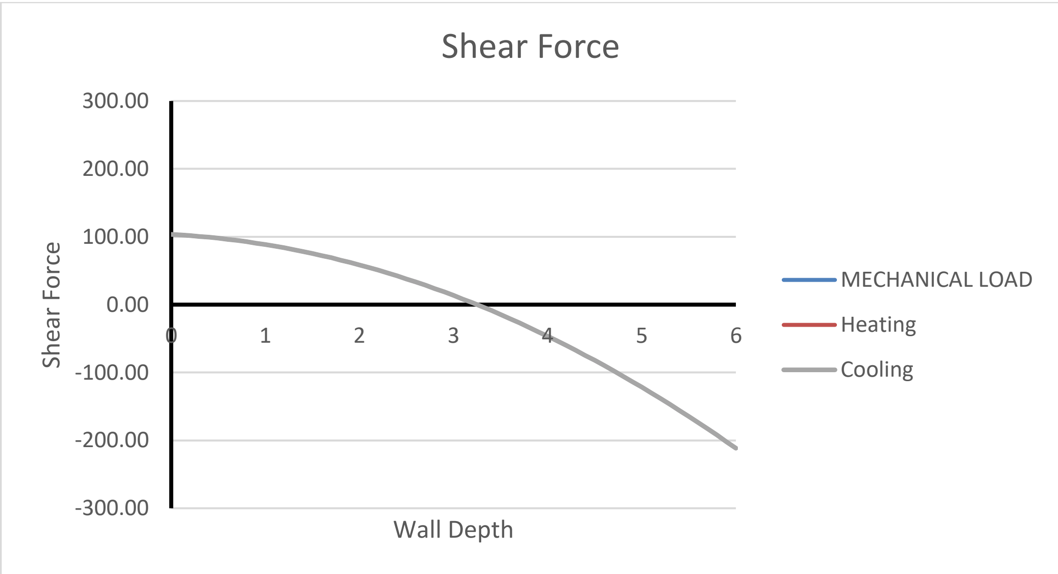




The temperature distribution during heating contributes to increase the positive bending moment M^+_{max}.

OVERALL COMPARISON





$$M_{max}^{+} = 151.73\ kN * m$$

$$M_{max}^{-} = -252\ kN * m$$

Thermal loads do not induce shear forces: $V_{max} = 204\ kN$